

Barrel length, velocity, and pressure

By 'tloc54' at *AmmoGuide*
Interactive

This article discusses velocity vs barrel length, which is old hat to some people, and adds a new twist, which is pressure vs barrel length. The pressure is derived in part from the acceleration of the bullet (the other part being the drag of the bullet in the bore). The magnitude of the acceleration may surprise you. Pressure is interesting inasmuch as it tells us what the minimum barrel strength must be along the barrel's length. More importantly, perhaps, is that the pressure at the time the bullet uncorks from the barrel is the major factor in the sharpness of the sound. The sharpness of the sound will be important if you are hunting in an area with more population than you'd like, and it will also be important for hearing damage.

Many handloaders are familiar with the Le Duc equation, which is a very simple equation relating bullet velocity to barrel length: change barrel length this much, and velocity changes that much. Although it is not theoretically exact, it gives amazingly good results (accurate within a few fps) for firearms over a wide range of expansion ratios. It occurred to me that I could use this equation to find the pressure of the powder gasses at the time the bullet exits the muzzle.

The volume of high pressure gas behind the bullet at the time the bullet exits is easy to find. However, in my experience, the sharpness of the sound has more to do with the pressure of the gas rather than its volume, witness a SAAMI standard pressure 45 Colt with any reasonable barrel length has a very dull sound, even though its gas volume is high.

I recently switched from using my new Ruger Mk III with 5 1/2" bull barrel to my old Ruger Mk II with 6 7/8" barrel (both using slightly subsonic match ammo) because the longer barrel is far friendlier to my severely-ringing ears: with proper ear protection, I don't get temporarily increased ringing. Since these barrel lengths are not so far different, they should make a perfect test case using Le Duc to define my personal maximum muzzle pressure.

I'll first review the Le Duc equation for those who are not familiar with it, then show how to get bullet acceleration out of it, and from that, pressure.

Review of the Le Duc equation

The equation is simple:

$$v = V_{max} * [x / (x + 2 * X_{mp})]$$

where

v is the muzzle velocity, measured in fps

x is how far the bullet base moves from rest until it exits the muzzle, measured in inches

V_{max} is the velocity the bullet would reach if the barrel were ridiculously long, measured in fps

X_{mp} is how far the bullet base has moved from rest when the pressure peaks, measured in inches

Let's apply it. Since we have two unknown constants, V_{max} and X_{mp}, we need to have two known answers to determine these constants. For SK Match 22 ammo, the velocity from a 6 7/8" barrel comes out 987 fps with a low standard deviation. From a 22" barrel, the velocity is very close to 1050 fps. Do the math, and find that:

Vmax = 1080 fps
Xmp = 0.3"

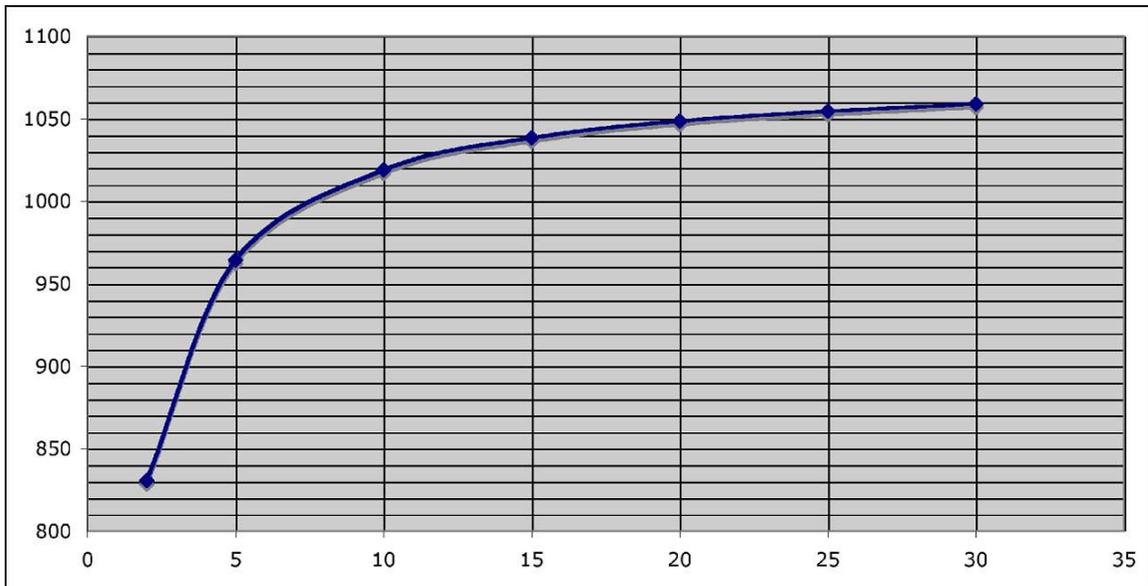
Let's check it out. The SK Match cartridge has 1/2" from the case head to the bullet base. Therefore a 6 7/8" barrel gives 6 3/8" of travel, x, while a 22" barrel gives x = 21.5".

For the 6 7/8" bbl (6 3/8" travel):
 $v = 1080 * [(6 \frac{3}{8}) / (6 \frac{3}{8} + 2*0.3)] = 987 \text{ fps}$
check

For the 22" bbl (21.5" travel):
 $v = 1080 * [(21.5) / (21.5 + 2*0.3)] = 1051 \text{ fps}$
check

Now we've learned that for this ammo and decent barrels, probably from about 2" length to about 26" length,
muzzle velocity = $1080 * [(\text{bullet travel}) / (\text{bullet travel} + 0.6)]$

You can cut your own constants for any cartridge with any load just by getting muzzle velocity for two different barrel lengths. The barrels must be of similar quality and similarly chambered, or you may use one barrel and shorten it a lot, taking measurements before and after shortening.



Here's a graph of velocity in fps vs bullet travel in inches for the SK Match

Finding acceleration

Velocity is measured in distance per unit time such as fps (feet per second). Acceleration is how fast velocity changes per unit time so it might be measured in fps per second. As an example, the "acceleration" of standard gravity is 32.174 fps/sec. Since the velocity of the bullet was zero when it started, and some large number of fps a short time later when it left the muzzle, it must have accelerated.

As an example, a bullet might accelerate from zero to a muzzle velocity of 2000 fps in .001 seconds, in which case its average acceleration was $2000 / .001 = 2,000,000$ fps/second which is 62,000 times the acceleration of gravity. Wow! But it gets worse. We know that the acceleration is highest near the beginning of the bullet's travel when the pressure is highest. (Pressure starts dropping off soon after the bullet starts moving.) 2,000,000

was just the average; it had to have been higher at the beginning and lower towards the end of its time in the barrel.

We have the equation for the velocity of the bullet as it travels down the barrel, and we can find the acceleration as it travels down the barrel by a mathematical trick called differentiation, where essentially you divide the equation for velocity by time in a mathematically exact way. The result is:

$$a = V_{max}^2 * 2 * X_{mp} * x / (2 * X_{mp} + x)^3$$

a is the acceleration, measured in inches per second per second

x is how far the bullet base moves from rest until it exits the muzzle, measured in inches

V_{max} is the velocity the bullet would reach if the barrel were ridiculously long, measured in inches per second

X_{mp} is how far the bullet base has moved from rest when the pressure peaks, measured in inches

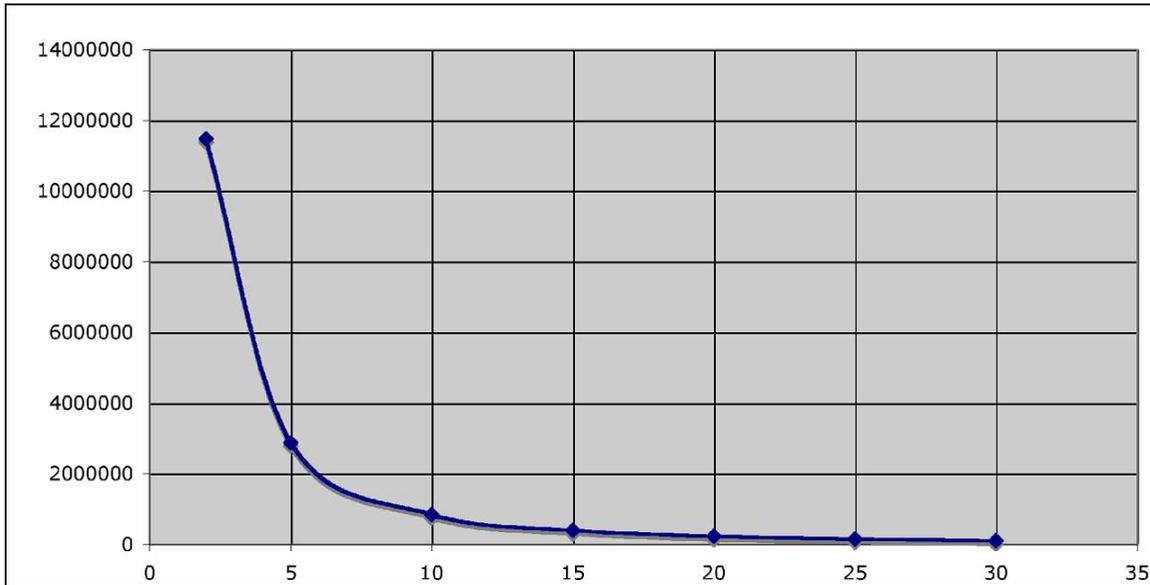
Notice how velocity is now in inches per second instead of fps? That's because after the differentiation, the units got all mixed together and so now we have to pick one set of units. I picked inches because I'd rather multiply 1000 fps by 12 to get 12,000 inches per second than divide 21.5 inches of barrel travel by 12 to get 1.7917 feet. Either way would work fine, though. Later, we can convert the answer to any units we like if we want to.

What is the acceleration of the bullet at the time it leaves the 5.5" barrel (has traveled 5")? What is the acceleration of the bullet at the time it has traveled 6 3/8 inches? How about for 21.5" of travel?

$a = V_{max}^2 * 2 * X_{mp} * x / (2 * X_{mp} + x)^3$
 $x = 5, 6.375, 21.5$ inches
 $V_{max} = 1080 * 12 = 12,960$ inches/second
 $X_{mp} = 0.3$ inches

from which,
 $a = 2,869,242$ inches/second/second for 5" travel
 $a = 1,893,254$ inches/second/second for 6.375" travel
 $a = 200,735$ inches/second/second for 21.5" travel

From this, we see acceleration is falling as the bullet moves down the barrel, but we knew that because we know pressure is falling.



Here's a graph of acceleration in inches/second squared vs bullet travel in inches

Finding force from acceleration

Remember that equation our teachers were trying to hammer into our heads in junior high school science class?

force = mass times acceleration

Well, we have acceleration, and we have the mass of the bullet, which in the English system is the bullet's weight divided by gravity. So now we can calculate the force. The bullet in the SK Match cartridge is supposed to weigh around 39 grains, but on my scales it comes closer to 40, so that's what I'll use. Since we've switched to inches to be consistent, we now have to use gravity in inches. $32.174 \text{ fps/sec} = 386.1 \text{ inches/sec/sec}$. Also, to use only one consistent unit, let's switch to pounds. $40 \text{ grains}/7000 = .0057 \text{ pounds}$. Now we can calculate the mass of our bullet in English units:

$$\begin{aligned} \text{mass} &= \text{weight}/\text{gravity} \\ &= .0057 \text{ pounds} / 386.1 \text{ inches/sec/sec} \\ &= 14.8/1,000,000 \text{ lb-sec}^2/\text{inch} \end{aligned}$$

Weird unit for mass, huh? Use it for forty-odd years, and it will seem quite normal. Better option today, learn metric.

OK, now we have the mass of the bullet, and we have the acceleration of the bullet from the previous section, so we can use the force equation to find the accelerating force on the bullet.

$$\text{force} = \text{mass times acceleration}$$

At 5" of travel:

$$\text{force} = (14.8/1,000,000) * 2,869,242 = 42.5 \text{ lb}$$

At 6.375" of travel:

$$\text{force} = (14.8/1,000,000) * 1,893,254 = 28.0 \text{ lb}$$

At 21.5" of travel:

$$\text{force} = (14.8/1,000,000) * 200,735 = 3.0 \text{ lb}$$

Pressure from force

Pressure is force divided by area. That's why pressure is often measured in pounds per square inch or psi. The cross-sectional area of the rifled bore is roughly the area of a circle .220"

in diameter.

$$\text{area} = (\pi/4) * .22^2 = .038 \text{ square inches}$$

Above, we've calculated the force on the bullet required to cause the acceleration it is experiencing, but I know from previous experiments that because of friction, you have to have about 25 pounds of force on a 22 lead bullet just to keep it moving down the bore. This is after the bullet's been engraved to fit the rifling. Therefore, we have to add 25 lbs of friction force to the accelerating force to find the total force on the base of the bullet.

At 5" of travel:

$$\text{force} = 42.5 \text{ lb} + 25 \text{ lb} = 67.5 \text{ lb}$$

At 6.375" of travel:

$$\text{force} = 28.0 \text{ lb} + 25 \text{ lb} = 53 \text{ lb}$$

At 21.5" of travel:

$$\text{force} = 3.0 \text{ lb} + 25 \text{ lb} = 28 \text{ lb}$$

As you can see, after the 22 long rifle bullet has moved down the barrel a little ways, most of the pressure in the barrel is being used to overcome the friction of the bullet in the barrel.

What's the pressure required to both accelerate the bullet and overcome friction?

pressure = force divided by area

At 5" of travel:

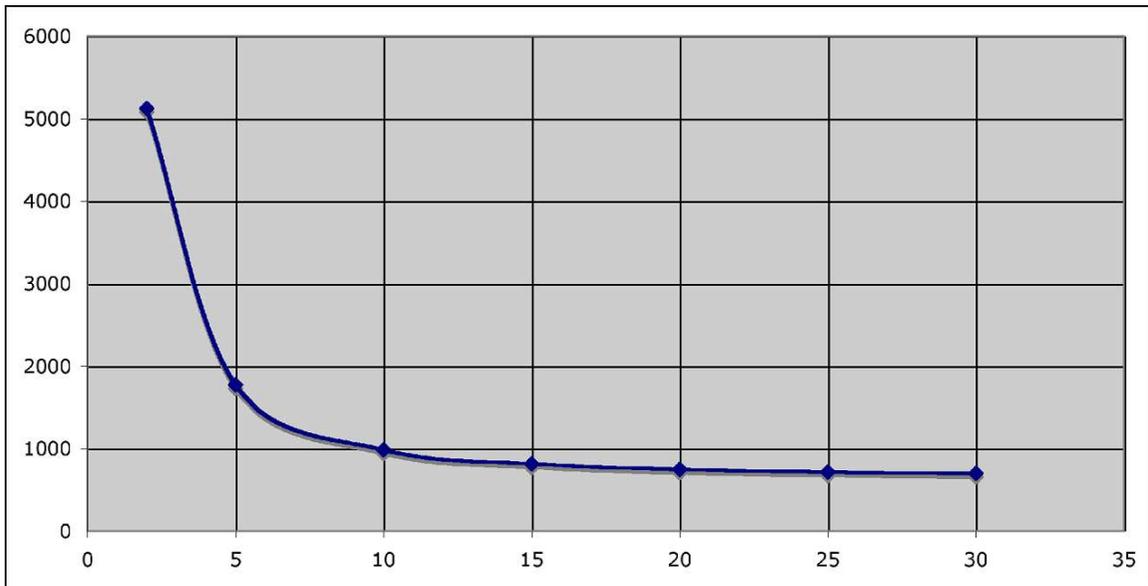
$$\text{pressure} = 67.5 \text{ lb} / .038 \text{ square inches} = 1776 \text{ psi}$$

At 6.375" of travel:

$$\text{pressure} = 53 \text{ lb} / .038 \text{ square inches} = 1395 \text{ psi}$$

At 21.5" of travel:

$$\text{pressure} = 28 \text{ lb} / .038 \text{ square inches} = 737 \text{ psi}$$



Here's a graph of pressure in psi vs bullet travel in inches

Conclusions

From the graph, we can see that if it is "be kind to the neighbors" week, a barrel of 10" or more is going to make a lot duller noise with this cartridge.

It seems that hearing-protected ears may strongly prefer a muzzle pressure below a particular value, a value which may be determined by experiment. From the calculations, I see that my ears strongly prefer 1400 psi to 1800 psi.

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